

# General nonextremal rotating charged AdS black holes in five-dimensional $U(1)^3$ gauged supergravity: A simple construction method

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## Abstract

With the help of a generalized form of the metric ansatz found for the single-charge case in a previous work [S.Q. Wu, Phys. Rev. D **83**, 121502(R) (2011)], I adopt a simple algorithm to construct the most general nonextremal rotating charged black hole solutions in five-dimensional  $U(1)^3$  gauged supergravity. The general solution that is interesting for testing the AdS<sub>5</sub>/CFT<sub>4</sub> correspondence in M-theory, is characterized by its mass, two unequal rotation parameters, three different  $U(1)$  charges, and a negative cosmological constant. The metric ansatz is very universal and illuminative, it is not only especially suitable for constructing solutions with multiple different electric charges in (un)gauged supergravities, but also for other dilatonic gravity theory.

*Keywords:* black hole, AdS<sub>5</sub>, gauged supergravity

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## 1. Introduction

The discovery of the anti-de Sitter/conformal field theory correspondence has stimulated a great deal of interest in the construction of rotating charged solutions in gauged supergravities. Of particular interest are five-dimensional nonextremal rotating charged anti-de Sitter (AdS) black holes within the maximal  $SO(6)$ -gauged  $D = 5$ ,  $\mathcal{N} = 8$  supergravity, since this theory arises from the  $S^5$  reduction of the type IIB superstring. Black holes with Abelian gauge fields can carry three independent charges, associated with the Cartan subgroup  $U(1)^3$  of  $SO(6)$ . Equivalently, they can be viewed as solutions in

$D = 5$ ,  $\mathcal{N} = 2$  gauged supergravity coupled to two additional vector multiplets. The general black hole solution should be characterized by its mass, two independent angular momenta, and three different charge parameters.

However, constructing such nonextremal rotating charged black hole solutions in  $D = 5$   $U(1)^3$  gauged supergravity becomes quite a difficult problem, since unlike the case of ungauged supergravity, there is no known solution-generating technique that could charge up the already-known neutral rotating  $AdS_5$  black hole solution found in [1]. On the other hand, it is also quite different from the case of supersymmetric solutions that receive considerable studies using the G-structure formalism which was introduced first in  $D = 5$  minimal gauged supergravity by [2, 3] and then in  $\mathcal{N} = 2$  gauged five-dimensional supergravity in [4, 5]. The basis for this technique relies on solving the Killing spinor equation [6, 7]. Nevertheless, supersymmetric  $AdS_5$  solutions can also have been successfully found as limits of nonextremal solutions without using such a G-structure formalism.

Although there has been much progress over the last few years in obtaining new, nonextremal, asymptotically  $AdS_5$  black hole solutions of five-dimensional gauged supergravity theory, one has not yet obtained a most general nonextremal solution with two rotation and three charge parameters arbitrary. Apart from a known supersymmetric  $AdS_5$  solution [8] with two independent rotation and three unequal charge parameters, all the nonextremal black hole families of exact rotating and charged solutions currently-found in  $D = 5$   $U(1)^3$  gauged supergravity theory have been obtained via applying two simplification strategies [9]: either setting two angular momenta equal or setting certain charges equal. Since in these two kinds of special cases, one could rely on an inspired ansatz to substantially minimize the difficulty. So far, all the previously-obtained solutions for rotating charged  $AdS$  black holes were not derived via a universal method other than via a combination of guesswork, followed by explicit brute-force verifications of the field equations.

The currently-known nonextremal rotating charged black hole solutions in the five-dimensional minimal and  $U(1)^3$  gauged supergravities are as follows. Apart from a supersymmetric  $AdS_5$  black hole solution [10], a general nonextremal rotating charged black hole solution [11] within  $D = 5$  minimal gauged supergravity was obtained soon after the discovery of its counterpart with two equal angular momenta [12]. On the other hand, for the most interesting case of  $U(1)^3$  gauged supergravity, all the presently-obtained nonextremal rotating charged  $AdS_5$  black hole solutions are limited to the

two special cases: either with some charges equal, or with equal rotation parameters. In the much simpler situation where two rotation parameters are set equal, the solution with three independent charges was obtained in [13]. For black holes with two independent rotation parameters, a nonextremal solution where two charges are equal but the third one is set to zero was found in [14]. This solution was then extended [15] to the case in which two of the three charges are set equal, with the third non-vanishing. In addition, a solution with only one charge non-zero was constructed in [16], extending the special case given in [14] with only one rotation parameter. The latter solution can be also viewed as that of Kaluza-Klein gauged supergravity in which a different form for the single-charge solution was recently presented in [17].

Although these remarkable progresses have been made during the past years, the most general nonextremal solution for a five-dimensional rotating  $\text{AdS}_5$  black hole with two arbitrary rotation parameters and three independent charges remains unknown until this work, since no solution-generating technique in gauged supergravity is available for deriving the nonextremal rotating charged  $\text{AdS}_5$  black hole solutions from the neutral Kerr- $\text{AdS}_5$  solution [1]. Instead one has little option but to resort to brute-force calculations, starting from a guessed ansatz in the special cases either with two equal rotation parameters or with two or three charges equal to verify that all the equations of motion are completely satisfied.

The main purpose of this work is two-fold. First, I propose a universal ansatz to construct general nonextremal rotating charged AdS black hole solutions in gauged supergravities. The metric ansatz essentially generalizes the one previously put forward in [17] for the single-charge case in all higher dimensions. It is especially convenient for constructing black hole solutions with multiple different electric charges in gauged and ungauged supergravity theories, but needless limited to such theories. Second, I construct a general nonextremal solution for a rotating charged black hole in five-dimensional gauged supergravity with two unequal angular momenta and with three independent charge parameters. This finalizes the long-unsolved question of seeking the most interesting nonextremal  $\text{AdS}_5$  black hole solution in five-dimensional  $U(1)^3$  gauged supergravity. After that, the conserved charges associated with the first law of thermodynamics are computed. Because of the complexity of the solution, a more detailed version is expected to discuss other interesting issues related to it.

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## 2. Kerr-AdS<sub>5</sub> and three-charge Cvetič-Youm solutions

To construct the most general nonextremal rotating charged AdS<sub>5</sub> black hole solution in  $U(1)^3$  gauged supergravity, it is instructive to present the solution in an appropriate form to which the already-known Kerr-AdS<sub>5</sub> vacuum solution [1] and the three-charge Cvetič-Youm solution [18, 19] degenerate, respectively, in the uncharged case and in the ungauged case. In terms of the Boyer-Lindquist coordinates, the Kerr-AdS<sub>5</sub> black hole [1] in a non-rotating frame at infinity can be conveniently written as

$$\begin{aligned}
ds^2 = & -\frac{(1+g^2r^2)\Delta_\theta}{\chi_a\chi_b} dt^2 + \Sigma \left( \frac{r^2 dr^2}{\tilde{\Delta}_r} + \frac{d\theta^2}{\Delta_\theta} \right) \\
& + \frac{(r^2+a^2)\sin^2\theta}{\chi_a} d\phi^2 + \frac{(r^2+b^2)\cos^2\theta}{\chi_b} d\psi^2 \\
& + \frac{2m}{\Sigma} \left( \frac{\Delta_\theta}{\chi_a\chi_b} dt - \frac{a\sin^2\theta}{\chi_a} d\phi - \frac{b\cos^2\theta}{\chi_b} d\psi \right)^2, \quad (1)
\end{aligned}$$

in which

$$\begin{aligned}
\tilde{\Delta}_r &= (r^2+a^2)(r^2+b^2)(1+g^2r^2) - 2mr^2, \\
\Delta_\theta &= 1 - g^2p^2, \quad \Sigma = r^2 + p^2, \quad p^2 = a^2\cos^2\theta + b^2\sin^2\theta, \\
\chi_a &= 1 - g^2a^2, \quad \chi_b = 1 - g^2b^2.
\end{aligned}$$

It is well-known that the above metric (1) can be put in the Kerr-Schild form after performing the coordinate transformations [20].

Now let us turn to the three-charge Cvetič-Youm solution [18, 19]. It is remarkable to find that its metric can be expressed in the following suggestive form

$$ds^2 = (H_1 H_2 H_3)^{1/3} \left[ -dt^2 + \frac{\Sigma r^2}{\bar{\Delta}_r} dr^2 + \Sigma d\theta^2 + (r^2 + a^2) \sin^2 \theta d\phi^2 \right. \\ \left. + (r^2 + b^2) \cos^2 \theta d\psi^2 + \frac{2ms_1^2}{\Sigma H_1 (s_1^2 - s_2^2)(s_1^2 - s_3^2)} k_1^2 \right. \\ \left. + \frac{2ms_2^2}{\Sigma H_2 (s_2^2 - s_1^2)(s_2^2 - s_3^2)} k_2^2 + \frac{2ms_3^2}{\Sigma H_3 (s_3^2 - s_1^2)(s_3^2 - s_2^2)} k_3^2 \right], \quad (2)$$

$$k_i = \frac{s_i c_1 c_2 c_3}{c_i} \left( \frac{c_i^2}{c_1 c_2 c_3} dt - a \sin^2 \theta d\phi - b \cos^2 \theta d\psi \right) \\ + \frac{c_i s_1 s_2 s_3}{s_i} \left( b \sin^2 \theta d\phi + a \cos^2 \theta d\psi \right), \quad (3)$$

where

$$\bar{\Delta}_r = (r^2 + a^2)(r^2 + b^2) - 2mr^2, \quad H_i = 1 + \frac{2ms_i^2}{\Sigma}.$$

Three  $U(1)$  Abelian gauge fields are given by  $A_i = 2m/(\Sigma H_i)k_i$ . Here and hereafter, the short notations  $c_i = \cosh \delta_i$  and  $s_i = \sinh \delta_i$  ( $i = 1, 2, 3$ ) are used.

It should be pointed out that the line element (2) is essentially a generalized form of the metric ansatz previously-presented in [17] for the single-charge case of the Kaluza-Klein AdS black holes in all higher dimensions. To see it more apparently, one can make coordinate transformations so that the metric (2) is re-expressed in terms of the Kerr-Schild coordinates, see the Appendix for details. In doing so, I find the metric tensor then can be written as

$$g_{\mu\nu} = (H_1 H_2 H_3)^{1/3} \left[ \eta_{\mu\nu} + \frac{2ms_1^2}{\Sigma H_1 (s_1^2 - s_2^2)(s_1^2 - s_3^2)} \bar{k}_{1\mu} \bar{k}_{1\nu} \right. \\ \left. + \frac{2ms_2^2}{\Sigma H_2 (s_2^2 - s_1^2)(s_2^2 - s_3^2)} \bar{k}_{2\mu} \bar{k}_{2\nu} + \frac{2ms_3^2}{\Sigma H_3 (s_3^2 - s_1^2)(s_3^2 - s_2^2)} \bar{k}_{3\mu} \bar{k}_{3\nu} \right], \quad (4)$$

and accordingly the three  $U(1)$  gauge potentials are  $A_i = 2m/(\Sigma H_i)\bar{k}_i$  modulo radial gauge transformations. In practice, one finds that it is more ef-

ficient to work with the line element in terms of the Boyer-Lindquist coordinates rather than using the Kerr-Schild coordinates since this will avoid doing some unnecessary and complicated coordinate transformations.

Now if one of the three charges is set to zero, one obtains the two-charge Cvetič-Youm solution in five dimensions. In arbitrary dimensions, the two-charge Cvetič-Youm black hole solutions [21] can be recast into a similar form which is given in Appendix A of [22]. For cases with more charges, one can proceed in a similar manner. Therefore, one reaches to a remarkable conclusion that all of already-known supergravity solutions with multiple different electric charges in ungauged theories have a universal metric structure that can be rewritten in the ansatz proposed in this way.

Having revealed the underlying metric structure shared by the black hole solutions in ungauged supergravity theories, then what is the case of gauged supergravity solutions? It is well-established that in the vacuum cases, one can simply replace the flat metric  $\eta_{\mu\nu}$  in the Kerr-Schild form by the AdS metric. Does this still work well in the gauged supergravity cases? In a previous work [17] that only dealt with the single-charge case in Kaluza-Klein supergravity, it was demonstrated this indeed is the case. What is more, it has been shown there [17] that the same holds also true for the cases of nonrotating and rotating charged AdS black holes with multiple pure electric charges in gauged supergravity theories, including the recently-found single-rotation solution [22] in  $D = 7$  dimensions and the  $D = 4$  two-charge solution [23]. In particular, limited to the interesting  $D = 5$  case, there is no exception for the gauged supergravity solution with three independent charges obtained in [13] in the case where the two rotation parameters are set equal.

These remarkable facts demonstrate that all previously-known supergravity black hole solutions with multiple different electric charges can be recast into a unified metric ansatz, regardless they belong to ungauged theories or gauged ones. In other words, supergravity black hole solutions in gauged theory inherit the same underlying metric structure as their ungauged counterparts. This significant feature of supergravity black hole solutions had not been exploited in any other previous work. Clearly, once an exact solution in ungauged supergravity has been found and recast into the ansatz given above, one can immediately replace the flat metric by the AdS metric to try the corresponding gauged supergravity solution. [The remaining task is just to determine the independent vectors  $k_i$ . However, if additional form fields with higher spin are excited, the computation is still extraordinarily

complicated.]

Therefore, it is suggested that the ansatz proposed here provides a universal method to construct the most general rotating charged AdS black hole solutions with multiple pure electric charges in gauged supergravity theories. This consists of one of the main results of this work. Apart from the example given in [22], here I will provide one more example that finalizes the goal for seeking the most interesting nonextremal AdS<sub>5</sub> black hole solution in five-dimensional  $U(1)^3$  gauged supergravity. With the guidance of the above ansatz, the explicit expression for the most general charged rotating AdS<sub>5</sub> solution with three unequal charges and with two independent rotation parameters has been successfully found here using a simple construction procedure.

### 3. General solution

Before presenting the new exact solution, it is now in a position to first display the theory of five-dimensional  $\mathcal{N} = 2$  gauged supergravity coupled to two vector multiplets. The Lagrangian for the bosonic sector is given by

$$\begin{aligned} \mathcal{L} = & \sqrt{-g} \left[ R + 4g^2(X_1 + X_2 + X_3) - 3(\partial\varphi_1)^2 - (\partial\varphi_2)^2 - \frac{1}{4} \sum_{i=1}^3 X_i^2 F_i^2 \right] \\ & + \frac{1}{4} \varepsilon^{\mu\nu\rho\sigma\lambda} F_{1\mu\nu} F_{2\rho\sigma} A_{3\lambda} , \end{aligned} \quad (5)$$

where  $g$  is the gauge-coupling constant, and the quantities  $X_i = H_i/(H_1 H_2 H_3)^{1/3}$  are formed from the two scalar fields  $\varphi_1$  and  $\varphi_2$  in the vector multiplets:

$$X_1 = e^{\varphi_1 + \varphi_2} , \quad X_2 = e^{\varphi_1 - \varphi_2} , \quad X_3 = e^{-2\varphi_1} . \quad (6)$$

To find the most general nonextremal solution in this theory, it is reasonable to first assume that the metric of the new exact solution has the

exquisite form

$$\begin{aligned}
ds^2 = & (H_1 H_2 H_3)^{1/3} \left[ - \frac{(1 + g^2 r^2) \Delta_\theta}{\chi_a \chi_b} dt^2 + \Sigma \left( \frac{r^2 dr^2}{\Delta_r} + \frac{d\theta^2}{\Delta_\theta} \right) \right. \\
& + \frac{(r^2 + a^2) \sin^2 \theta}{\chi_a} d\phi^2 + \frac{(r^2 + b^2) \cos^2 \theta}{\chi_b} d\psi^2 \\
& + \frac{2ms_1^2}{\Sigma H_1 (s_1^2 - s_2^2)(s_1^2 - s_3^2)} K_1^2 + \frac{2ms_2^2}{\Sigma H_2 (s_2^2 - s_1^2)(s_2^2 - s_3^2)} K_2^2 \\
& \left. + \frac{2ms_3^2}{\Sigma H_3 (s_3^2 - s_1^2)(s_3^2 - s_2^2)} K_3^2 \right], \tag{7}
\end{aligned}$$

and the three  $U(1)$  Abelian gauge potentials are given by  $A_i = 2m/(\Sigma H_i) K_i$ . This is simply because the already-known equal-rotation ( $a = b$ ) solution [13] can be written in such a form, without any doubt, there should be no exception for the general case with two unequal rotations ( $a \neq b$ ). Then the remaining thing is to determine three unknown vectors  $K_i$  and the radial unction  $\Delta_r$ , since all other expressions remain unchanged. There are two different ways to arrive at this goal. One direct method is to analytically solve all the equations of motion derived from the Lagrangian after substituting the above metric ansatz and three gauge potentials into them. However, the subsequent computation will be very involved. Instead, below I will present another simple algorithm to implement this aim.

The construction procedure for the explicit solution consists of the following four steps: (I) Write down the conjectured metric (7) using the ansatz proposed above; (II) Construct three independent vectors  $K_i$  using a lot of rules (see below); (III) Determine the radial function  $\Delta_r$ ; (IV) Verify that all the field equations are completely satisfied. The first step has been completed in the above.

The most key point of this construction is how to write down the components of three vectors  $K_i$ , which is summarized according to my previous experience as follows. (i) First, combining the uncharged case (1) and the ungauged case (3), the expressions of three  $K_i$  are conjectured to behave like

$$\begin{aligned}
K_i \sim & C_{1i} \frac{s_i c_1 c_2 c_3}{c_i} \left( \frac{c_i^2}{c_1 c_2 c_3} \frac{\Delta_\theta}{\chi_a \chi_b} dt - C_{2i} \frac{a \sin^2 \theta}{\chi_a} d\phi - C_{3i} \frac{b \cos^2 \theta}{\chi_b} d\psi \right) \\
& - C_{4i} \frac{g^2 ab \Delta_\theta}{\chi_a \chi_b} dt + \frac{c_i s_1 s_2 s_3}{s_i} \left( C_{5i} \frac{b \sin^2 \theta}{\chi_a} d\phi + C_{6i} \frac{a \cos^2 \theta}{\chi_b} d\psi \right), \tag{8}
\end{aligned}$$



since the expected new solution should include them as two special cases. The 18 undetermined constants  $(C_{1i}, \dots, C_{6i})$  will be constructed in the next step. (ii) Second, using the discrete inversion symmetry

$$\begin{aligned} a &\rightarrow \frac{1}{ag^2}, \quad b \rightarrow \frac{b}{ag}, \quad m \rightarrow \frac{m}{a^4g^4}, \quad s_i \rightarrow ags_i, \\ gt &\rightarrow \phi, \quad \phi \rightarrow gt, \quad \psi \rightarrow \psi, \quad r \rightarrow \frac{r}{ag}, \quad p \rightarrow \frac{p}{ag}, \end{aligned} \quad (9)$$

and the interchange symmetry ( $a \leftrightarrow b$  and  $\phi \leftrightarrow \psi$ ) to write down all 18 constants  $(C_{1i}, \dots, C_{6i})$ . Noting that  $c_i \rightarrow \sqrt{\Xi_{ia}}$  under the inversion symmetry (9), one can very easily obtain the explicit expressions of all 15 undetermined constants except the three ones  $C_{4i}$  since the fourth term in the conjectured expressions (8) is new, it did not appear in the two special cases mentioned before. However, there is no difficulty to determine them by using the above symmetries at this step. The final expressions for three vectors  $K_i$  are found to be

$$\begin{aligned} K_i = & \frac{s_i c_1 c_2 c_3}{c_i} \frac{\sqrt{\Xi_{1a} \Xi_{2a} \Xi_{3a} \Xi_{1b} \Xi_{2b} \Xi_{3b}}}{\sqrt{\Xi_{ia} \Xi_{ib}}} \left( \frac{c_i^2}{c_1 c_2 c_3} \frac{\Delta_\theta}{\chi_a \chi_b} dt \right. \\ & - \frac{\Xi_{ia}}{\sqrt{\Xi_{1a} \Xi_{2a} \Xi_{3a}}} \frac{a \sin^2 \theta}{\chi_a} d\phi - \frac{\Xi_{ib}}{\sqrt{\Xi_{1b} \Xi_{2b} \Xi_{3b}}} \frac{b \cos^2 \theta}{\chi_b} d\psi \Big) \\ & + \frac{c_i s_1 s_2 s_3}{s_i} \sqrt{\Xi_{ia} \Xi_{ib}} \left( - \frac{c_1 c_2 c_3}{c_i^2} \frac{g^2 ab \Delta_\theta}{\chi_a \chi_b} dt \right. \\ & \left. + \frac{\sqrt{\Xi_{1a} \Xi_{2a} \Xi_{3a}}}{\Xi_{ia}} \frac{b \sin^2 \theta}{\chi_a} d\phi + \frac{\sqrt{\Xi_{1b} \Xi_{2b} \Xi_{3b}}}{\Xi_{ib}} \frac{a \cos^2 \theta}{\chi_b} d\psi \right), \end{aligned} \quad (10)$$

in which  $\Xi_{ia} = c_i^2 - s_i^2 \chi_a$  and  $\Xi_{ib} = c_i^2 - s_i^2 \chi_b$ .

At this time, it should be pointed out that the above construction of the vectors  $K_i$  are largely helped by the discovery of the two-charge solution [24] in the theory and the single-charge solution [17] in the Kaluza-Klein theory. It is then realized that the above-mentioned discrete inversion symmetry (9) and the interchange symmetry play a crucial role in the construction of the new exact solution. It should be mentioned that the inversion symmetry (9) was first observed in [25] for the uncharged Kerr-AdS<sub>5</sub> solution, and then in the  $D = 4$  rotating charged solutions recently found by Chow [23, 26]. It should also note that the recently-found single-charge solutions [17] in all dimensions and the two-charge solution [22] in  $D = 7$  dimensions exhibit

the same discrete inversion symmetry. By the construction procedure, the general solution presented here obviously endows with the discrete inversion symmetry (9).

Having given the full expressions of all three vectors  $K_i$ , it is of no difficulty to obtain the explicit formulae of the radial function

$$\begin{aligned}\Delta_r = & (r^2 + a^2)(r^2 + b^2)(1 + g^2 r^2) - 2mr^2 + 2mg^2 \left\{ (s_1^2 + s_2^2 + s_3^2)r^4 \right. \\ & - (s_1^2 s_2^2 + s_1^2 s_3^2 + s_2^2 s_3^2) [(a^2 + b^2 - 2m)r^2 + a^2 b^2 (2 + g^2 r^2)] \\ & + s_1^2 s_2^2 s_3^2 \left[ ((a + b)^2 - 2m)[(a - b)^2 - 2m] - 2g^2 a^2 b^2 (2r^2 + 2m \right. \\ & \left. + a^2 + b^2) + g^4 a^4 b^4 \right] + 2mg^2 a^2 b^2 [s_1^4 s_2^4 + s_1^4 s_3^4 + s_2^4 s_3^4 \\ & \left. - 2s_1^2 s_2^2 s_3^2 (s_1^2 + s_2^2 + s_3^2)] \right\},\end{aligned}\tag{11}$$

by requiring the metric determinant to be

$$\sqrt{-g} = (H_1 H_2 H_3)^{1/3} \frac{\Sigma r \sin \theta \cos \theta}{\chi_a \chi_b}.\tag{12}$$

The final step is to mechanically verify that all of the field equations are indeed satisfied. This point can be confirmed by the fact that the new exact solution naturally degenerates to the uncharged case [1] and the ungauged case [18, 19], it also reduces to the  $D = 5$  single-charge case found in [17] and the two-charge case [24] in their corresponding limits. In all of these cases, the two rotation parameters are independently specified. When three  $U(1)$  charges are independent of each other, the verification has also been analytically implemented in the special case with just one rotation parameter and in the case when the two angular momenta are set equal. However, limited to the memory of a personal computer and the 32-byte Maple 7 program, the complete verification fails in the minimal supergravity case (with three equal charges and with two unequal rotation parameters) and in the most general case (with three different charges and with two unequal rotation parameters), due to the complexity of the new exact solution. Nevertheless, the verification of the most general solution can be very easily checked by choosing several groups of different numerical values of  $(m, g, a, b, \delta_1, \delta_2, \delta_3)$ .

The general solution constructed here is characterized by its mass, two unequal rotation parameters, three different  $U(1)$  charges, and a negative cosmological constant. It is interesting for the  $\text{AdS}_5/\text{CFT}_4$  correspondence in M-theory.

#### 4. Thermodynamics

Having presented the explicit expression of the new exact solution, the last task of this work is to examine its thermodynamics, leaving the other interesting issues to be further investigated in a detailed version. The general black hole solution has a Killing horizon at  $r = r_+$ , the largest positive root of  $\Delta_r = 0$ . On the outer horizon, the entropy and the Hawking temperature are easily evaluated as

$$S = \frac{\pi^2}{2\chi_a\chi_b r_+} \sqrt{W}, \quad T = \frac{\Delta'_{r_+}}{4\pi\sqrt{W}}, \quad (13)$$

where

$$\begin{aligned} W = & [(r_+^2 + a^2)(r_+^2 + b^2) + 2mr_+^2(s_1^2 + s_2^2 + s_3^2)] \left\{ (r_+^2 + a^2)(r_+^2 + b^2) \right. \\ & + 2mg^2[(a+b)^2 - g^2a^2b^2][(a-b)^2 - g^2a^2b^2]s_1^2s_2^2s_3^2 - 4mg^2a^2b^2(s_1^2s_2^2 \\ & + s_1^2s_3^2 + s_2^2s_3^2) \left. \right\} + 8m^2r_+^2c_1c_2c_3s_1s_2s_3ab\sqrt{\Xi_{1a}\Xi_{2a}\Xi_{3a}\Xi_{1b}\Xi_{2b}\Xi_{3b}} \\ & + 4m^2r_+^2[r_+^2 + g^2a^2b^2(s_1^2 + s_2^2 + s_3^2)](s_1^2s_2^2 + s_1^2s_3^2 + s_2^2s_3^2) \\ & - 4m^2 \left\{ (a^2 + b^2)(1 + g^2a^2)(1 + g^2b^2)r_+^2 + g^2[(a^4 + b^4)r_+^2 \right. \\ & + g^2a^4b^4(2 + g^2r_+^2)](s_1^2 + s_2^2 + s_3^2) + g^2a^2b^2(a^2 + b^2 \\ & + g^2a^2b^2)[2 + g^2r_+^2(s_1^2s_2^2 + s_1^2s_3^2 + s_2^2s_3^2)] \left. \right\} s_1^2s_2^2s_3^2 \\ & + 4m^2g^4a^4b^4(s_1^4s_2^4 + s_1^4s_3^4 + s_2^4s_3^4) - 8m^2r_+^2g^6a^4b^4s_1^4s_2^4s_3^4 \\ & + 8m^3(r_+^2 + g^2a^2b^2)s_1^2s_2^2s_3^2. \end{aligned}$$

The angular velocities at the horizon are given by

$$\begin{aligned}\Omega_a = & \frac{2mr_+^2}{W} \left\{ ac_1c_2c_3\sqrt{\Xi_{1a}\Xi_{2a}\Xi_{3a}} \left[ r_+^2 + b^2 - 2mg^2b^2(s_1^2s_2^2 + s_2^2s_3^2 \right. \right. \\ & + s_1^2s_3^2 + 2g^2b^2s_1^2s_2^2s_3^2) \Big] - bs_1s_2s_3\sqrt{\Xi_{1b}\Xi_{2b}\Xi_{3b}} \left[ (r_+^2 + b^2)\chi_a^2 \right. \\ & - 2m[1 + g^2a^2 + 2g^2a^2(s_1^2 + s_2^2 + s_3^2) + g^2a^2(s_1^2s_2^2 + s_2^2s_3^2 \\ & + s_1^2s_3^2)(1 + g^2a^2) + 2g^4a^4s_1^2s_2^2s_3^2] \Big] \Big\},\end{aligned}\quad (14)$$

$$\begin{aligned}\Omega_b = & \frac{2mr_+^2}{W} \left\{ bc_1c_2c_3\sqrt{\Xi_{1b}\Xi_{2b}\Xi_{3b}} \left[ r_+^2 + a^2 - 2mg^2a^2(s_1^2s_2^2 + s_2^2s_3^2 \right. \right. \\ & + s_1^2s_3^2 + 2g^2a^2s_1^2s_2^2s_3^2) \Big] - as_1s_2s_3\sqrt{\Xi_{1a}\Xi_{2a}\Xi_{3a}} \left[ (r_+^2 + a^2)\chi_b^2 \right. \\ & - 2m[1 + g^2b^2 + 2g^2b^2(s_1^2 + s_2^2 + s_3^2) + g^2b^2(s_1^2s_2^2 + s_2^2s_3^2 \\ & + s_1^2s_3^2)(1 + g^2b^2) + 2g^4b^4s_1^2s_2^2s_3^2] \Big] \Big\},\end{aligned}\quad (15)$$

while the three electrostatic potentials are computed as

$$\begin{aligned}\Phi_i = & \frac{2mr_+^2}{W} \left\{ \frac{c_is_i\sqrt{\Xi_{1a}\Xi_{2a}\Xi_{3a}\Xi_{1b}\Xi_{2b}\Xi_{3b}}}{\sqrt{\Xi_{ia}\Xi_{ib}}} \left[ (r_+^2 + a^2)(r_+^2 + b^2) + 2m(s_1^2 + s_2^2 \right. \right. \\ & + s_3^2 - s_i^2)(r_+^2 - g^2a^2b^2s_i^2) - 4mg^2a^2b^2s_1^2s_2^2s_3^2 - 2m(a^2 + b^2 \\ & - 2m)\frac{s_1^2s_2^2s_3^2}{s_i^2} \Big] - \frac{abc_1c_2c_3s_1s_2s_3\sqrt{\Xi_{ia}\Xi_{ib}}}{c_is_i} \left( g^2(r_+^2 + a^2)(r_+^2 + b^2) \right. \\ & - 2m(1 + 2s_i^2) + 2mg^2(s_1^2 + s_2^2 + s_3^2 - s_i^2)[r_+^2 - s_i^2(a^2 + b^2)] \\ & \left. \left. + 2mg^2[2m - (1 + 2s_i^2)g^2a^2b^2]\frac{s_1^2s_2^2s_3^2}{s_i^2} \right) \right\}.\end{aligned}\quad (16)$$

Clearly, the new exact solution is asymptotically AdS with the boundary metric

$$\lim_{r \rightarrow \infty} \frac{ds^2}{r^2} = -\frac{g^2\Delta_\theta}{\chi_a\chi_b} dt^2 + \frac{dr^2}{g^2r^4} + \frac{d\theta^2}{\Delta_\theta} + \frac{\sin^2\theta}{\chi_a} d\phi^2 + \frac{\cos^2\theta}{\chi_b} d\psi^2, \quad (17)$$

with which one can choose two vectors

$$\hat{N}^a = \frac{\sqrt{\chi_a\chi_b}}{g\sqrt{\Delta_\theta}}(\partial_t)^a, \quad \hat{n}^a = -g^2r^2(\partial_r)^a,$$

and use the procedure adopted in [27] to compute the conserved charges that obey thermodynamical first laws.

Using the formulae

$$\begin{aligned}\mathcal{Q}[\xi] &= \frac{1}{16\pi} \int_{S^3} d^3x \frac{\sin \theta \cos \theta}{g^3 \sqrt{\chi_a \chi_b \Delta_\theta}} C_{acbd} \xi^a \hat{N}^b \hat{n}^c \hat{n}^d \\ &= \frac{\pi}{4} \int_0^{\pi/2} d\theta \frac{\sin \theta \cos \theta}{\Delta_\theta} r^4 C_{artr} \xi^a, \end{aligned} \quad (18)$$

the conserved mass and the two angular momenta are computed as

$$\begin{aligned}M &= -\mathcal{Q}[\partial_t] = \frac{\pi m}{4\chi_a \chi_b} \left\{ 2c_1^2 c_2^2 c_3^2 \left( \frac{1}{\chi_a} + \frac{1}{\chi_b} - 1 \right) + 1 - (s_1^2 s_2^2 + s_2^2 s_3^2 \right. \\ &\quad \left. + s_1^2 s_3^2)(1 + \chi_a \chi_b) - 2s_1^2 s_2^2 s_3^2 [1 + (2 - \chi_a - \chi_b)\chi_a \chi_b] \right\}, \end{aligned} \quad (19)$$

$$J_a = \mathcal{Q}[\partial_\phi] = \frac{\pi m}{2\chi_a^2 \chi_b} (ac_1 c_2 c_3 \sqrt{\Xi_{1a} \Xi_{2a} \Xi_{3a}} - b\chi_a^2 s_1 s_2 s_3 \sqrt{\Xi_{1b} \Xi_{2b} \Xi_{3b}}), \quad (20)$$

$$J_b = \mathcal{Q}[\partial_\psi] = \frac{\pi m}{2\chi_a \chi_b^2} (bc_1 c_2 c_3 \sqrt{\Xi_{1b} \Xi_{2b} \Xi_{3b}} - a\chi_b^2 s_1 s_2 s_3 \sqrt{\Xi_{1a} \Xi_{2a} \Xi_{3a}}), \quad (21)$$

while three electric charges can be evaluated as

$$\begin{aligned}Q_i &= \frac{1}{16\pi} \int_{S^3} \left( X_i^{-2} {}^\star F_i - \frac{1}{2} \epsilon_{ijk} A_j \wedge F_k \right) \\ &= \frac{\pi m}{2\chi_a \chi_b} \left( \frac{c_i s_i \sqrt{\Xi_{1a} \Xi_{2a} \Xi_{3a} \Xi_{1b} \Xi_{2b} \Xi_{3b}}}{\sqrt{\Xi_{ia} \Xi_{ib}}} - g^2 ab \frac{c_1 c_2 c_3 s_1 s_2 s_3}{c_i s_i} \sqrt{\Xi_{ia} \Xi_{ib}} \right). \end{aligned} \quad (22)$$

These conserved charges are related by the first law of thermodynamics.

## 5. Conclusions

In this paper, I have proposed a universal ansatz that is not only especially suitable for constructing black hole solutions with multiple unequal electric charges in gauged and ungauged supergravity theories, but also for other dilatonic gravity theory. With the help of the ansatz, a simple algorithm is then adopted to successfully construct the most general nonextremal rotating charged AdS<sub>5</sub> black hole solution with two unequal angular momenta and with three different charge parameters within five-dimensional  $U(1)^3$  gauged supergravity. I have also computed the conserved charges associated with the first law of thermodynamics.

Because of the complexity of the results presented in this article, many other interesting issues leave to be further revealed. For example, a direct problem associated with the new solution is to investigate the relations between the two previously-known solutions [11, 13] and the special cases where three charges are set equal or two rotation parameters are set equal in the general solution presented here. On the other hand, one would like to seek the supersymmetric limit of the general solution, just as Ref. [8] did.

Furthermore, the metric ansatz proposed here is a most important development of the famous Kerr-Schild ansatz, it needs further deeper investigations in different supergravity theories. It is anticipated that the ansatz can open a new way towards constructing the most general rotating charged black hole solutions with multiple pure electric charges yet to be found in some other (un) gauged supergravity theories.

By the way, it is also interesting to ask whether the five-dimensional solutions with non-spherical topology structure such as black string, black ring and black Saturn can be recast into a similar ansatz. Another question is whether multi-black hole solutions and multi-black ring solutions can be written in a similar formalism. Clearly, these points deserve a deeper investigation since this will be greatly expand and deepen our knowledge in the construction of exact solutions in higher dimensions.

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## Appendix

In order to rewrite the Cvetič-Youm solution (2)

$$\begin{aligned}
ds^2 = & (H_1 H_2 H_3)^{1/3} \left\{ \frac{\Sigma r^2}{\bar{\Delta}_r} dr^2 + \Sigma d\theta^2 + (r^2 + a^2) \sin^2 \theta d\phi^2 \right. \\
& + (r^2 + b^2) \cos^2 \theta d\psi^2 - \frac{\Sigma - 2m}{\Sigma H_1 H_2 H_3} \left[ dt + \frac{2mc_1 c_2 c_3}{\Sigma - 2m} (a \sin^2 \theta d\phi \right. \\
& + b \cos^2 \theta d\psi) - \frac{2ms_1 s_2 s_3}{\Sigma} (b \sin^2 \theta d\phi + a \cos^2 \theta d\psi) \left. \right]^2 \\
& \left. + \frac{2m}{\Sigma - 2m} (a \sin^2 \theta d\phi + b \cos^2 \theta d\psi)^2 \right\}, \tag{A1}
\end{aligned}$$

in terms of the Kerr-Schild coordinates as follows

$$\begin{aligned}
ds^2 = & (H_1 H_2 H_3)^{1/3} \left[ -d\bar{t}^2 + \frac{\Sigma r^2}{(r^2 + a^2)(r^2 + b^2)} d\bar{r}^2 \right. \\
& + \Sigma d\theta^2 + (r^2 + a^2) \sin^2 \theta d\bar{\phi}^2 \\
& + (r^2 + b^2) \cos^2 \theta d\bar{\psi}^2 + \frac{2ms_1^2}{\Sigma H_1 (s_1^2 - s_2^2)(s_1^2 - s_3^2)} \bar{k}_1^2 \\
& \left. + \frac{2ms_2^2}{\Sigma H_2 (s_2^2 - s_1^2)(s_2^2 - s_3^2)} \bar{k}_2^2 + \frac{2ms_3^2}{\Sigma H_3 (s_3^2 - s_1^2)(s_3^2 - s_2^2)} \bar{k}_3^2 \right], \tag{A2}
\end{aligned}$$

in which

$$\begin{aligned}
\bar{k}_i = & \frac{s_i c_1 c_2 c_3}{c_i} \left[ \frac{c_i^2}{c_1 c_2 c_3} d\bar{t} - a \sin^2 \theta d\bar{\phi} - b \cos^2 \theta d\bar{\psi} + \frac{\Sigma r^2 dr}{(r^2 + a^2)(r^2 + b^2)} \right] \\
& + \frac{c_i s_1 s_2 s_3}{s_i} \left[ b \sin^2 \theta d\bar{\phi} + a \cos^2 \theta d\bar{\psi} - \frac{2\epsilon m a b \Sigma r^2 dr}{(r^2 + a^2)(r^2 + b^2) \bar{\Delta}_r} \right],
\end{aligned}$$

one can make the following coordinate transformations:

$$\begin{aligned}
dt = & d\bar{t} - \left[ c_1 c_2 c_3 + \frac{2(1 - \epsilon) m a b s_1 s_2 s_3}{(r^2 + a^2)(r^2 + b^2)} \right] \frac{2mr^2}{\bar{\Delta}_r} dr, \\
d\phi = & d\bar{\phi} - \frac{2amr^2 dr}{(r^2 + a^2) \bar{\Delta}_r}, \quad d\psi = d\bar{\psi} - \frac{2bmr^2 dr}{(r^2 + b^2) \bar{\Delta}_r}, \tag{A3}
\end{aligned}$$

where  $\epsilon$  takes 0, or 1.

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